

# Prices, Rents and Speculative Bubbles in the Sydney Housing Market\*

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## Abstract

From the mid-1990s to 2003-04 the price-rent ratio for residential property in Sydney approximately doubled. At the peak of the Sydney housing boom price-rent ratios indicated a willingness to pay of between 30 to 40 times annual rentals. In this paper we examine possible reasons for the increase in the price-rent ratio and whether it is likely to be sustained. Asset pricing theory provides three explanations for rises in price-rent ratios: anticipated rises in the growth rate of rents; anticipated falls in discount factors or a rational speculative bubble. Using price and rent data for Local Government Areas (LGAs) in Sydney, we find little evidence that the rise in price-rent ratios anticipated future growth in rents. Rather changes in price-rent ratios were most probably driven by changing expectations about future discount factors, in part, the long-term real interest rate. However for some of Sydney's outer western regions we cannot completely rule out the possibility the property prices were (are) affected by a rational speculative bubble.

**Keywords:** price-rent ratio, present-value model, discount factor, property prices, speculative bubble

**JEL Classification:** G12, R31

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## 1. Introduction

In the period from the mid-1990s to around 2003-04 the prices of houses and apartments in Sydney displayed strong growth. Associated with this period of growth in prices is a significant rise in the price-rent ratio for residential property. Figure 1 illustrates the rise in the price-rent ratio using data for established houses in Sydney over the period 1982 to 2005<sup>1</sup>. The price-rent ratio displays a number of local trends. Up until 1987 the price-rent ratio lies in the range 10-15 and actually displays a slight downward trend. There is a sharp rise in the price-rent ratio in the late 1980s after which it fluctuates in the range 15-20 for a number of years. From 1996 the price-rent ratio displays a persistent upward trend, sharply rising in 2001 and then peaking around 2003-04, after which it declines slightly. At the peak of the recent Sydney housing boom buyers were willing pay up to 40 times the annual market rent on a house. This price-rent ratio is close to double what buyers were willing to pay in the first half of the 1990s.

In this paper we look at possible explanations for the large increase in the Sydney house prices relative to rents from 1996 to 2004. In particular we are interested in the ability of standard asset pricing theory to explain the behaviour of the price-rent ratio. The behaviour of the price-rent ratio from the mid-1990s is reminiscent of the stock price-earnings (or price-dividend) ratio in the United States over the last part of the 20 century (Shiller, 2005). The models used in this paper have been widely applied to stock markets and to understanding the behaviour of the price-dividend ratio for stocks. A review of the empirical evidence for stocks is proved by Cochrane (2001). While there are some obvious differences between stock markets and housing markets we are interested in examining how well asset price theory performs in explaining house prices. Even if standard asset pricing models fail to adequately explain the behaviour of the price-rent ratio do they fail in ways that are similar to the case of stock markets?

For some commentators the sharp rises in Sydney house prices in the years leading up to 2004 is *prima facie* evidence of a speculative bubble. Convincing empirical evidence of bubbles in asset markets has been hard to find as it is always possible to argue that some economic fundamental has not been taken into account. Subject to this caveat, the

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<sup>1</sup> The quarterly data for this figure are produced by the Real Estate Institute of Australia (REIA). The price series measures median established house prices and the rent series is the median annual rent on a three bedroom house.

model used in the paper does indicate whether or not it is necessary to resort to the existence of bubbles to explain the behaviour of the price-rent ratio.

Previous studies that analyse residential property markets from an asset pricing perspective typically fall into two categories. One group of papers have examined the predictability of returns (or excess returns) to housing (Case and Shiller, 1989; 1990; Hamilton and Schwab, 1985). A second group of studies make use of the present value relation linking house prices to economic fundamentals, typically discounted future expected rents (Meese and Wallace, 1994; Clayton, 1996; Ayuso and Restoy, 2003; 2006). Formal statistical tests typically lead to the rejection of the forward-looking present value model. However despite the statistical rejections, most studies find some informal support for the present value model, in particular there appears to be some medium to long-term relationship between house prices and rents. In this paper we follow the second approach.

The simplest and most restrictive asset pricing model for houses predicts that the price of a house should equal the present discounted value of the future stream of (explicit or implicit) rents. Furthermore when the discount rate is assumed to be a constant we expect (in the absence of a rational bubble) to observe a common trend in house prices and rents. It is immediately clear from Figure 1 that the simplest version of the present value model is very unlikely to provide a good explanation for the Sydney data, since the large and apparently persistent rise in the price-rent ratio must imply different (local) trends in house prices and rents.

In this paper we use a generalized version of the present-value relation that allows for a stochastic discount factor (Campbell and Shiller, 1988; Cochrane, 1992). This model has been widely applied to analysis of the stock price-dividend ratio resulting in some well-documented empirical anomalies (Cochrane, 2001). One objective of this paper is to see if similar anomalies are found for housing markets as arise for stock markets.

The structure of the paper is as follows. Section 2 outlines the asset pricing model that forms the basis for the empirical tests in this model. Section 3 describes the data used in this study and presents some summary statistics. Empirical results are presented in Section 4 and Section 5 concludes.

## 2. An Asset Pricing Model for the Price-Rent Ratio

### A Present-Value Model

Following Cochrane (1992) we begin with the general asset pricing equation,

$$1 = E_t[M_{t+1}(1 + r_{t+1}^h)] \quad (1)$$

where  $M_{t+1}$  is the stochastic discount factor,  $(1 + r_{t+1}^h) \equiv \frac{P_{t+1}^h + R_{t+1}^h}{P_t^h}$ ,  $P_t^h$  is the house price at

time  $t$ ,  $R_t^h$  is the rent on a house at time  $t$  and  $E_t$  is the expectation operator conditional on information at time  $t$ . Equation (1) can be iterated forward to obtain an equation for the price of a house;

$$P_t^h = E_t \sum_{i=1}^T \left( \prod_{k=1}^i M_{t+k} \right) R_{t+i}^h + E_t \left( \prod_{k=1}^T M_{t+k} \right) P_{t+T}^h \quad (2)$$

then if the transversality condition  $\lim_{T \rightarrow \infty} E_t \left[ \left( \prod_{k=1}^T M_{t+k} \right) P_{t+T}^h \right]$  is imposed to rule out rational

bubbles we obtain;

$$P_t^h = E_t \sum_{i=1}^{\infty} \left( \prod_{k=1}^i M_{t+k} \right) R_{t+i}^h \quad (3)$$

Equation (3) states that the current price of a house is equal to the expected present-discounted value of future rents.

Given our focus is on the price-rent ratio, we divide both sides of (3) by  $R_t^h$  to obtain

$$\frac{P_t^h}{R_t^h} = E_t \sum_{i=1}^{\infty} \left( \prod_{k=1}^i M_{t+k} \right) \frac{R_{t+i}^h}{R_t^h} \quad (4)$$

or

$$\frac{P_t^h}{R_t^h} = E_t \sum_{i=1}^{\infty} \left( \prod_{k=1}^i M_{t+k} G_{t+k}^R \right) \quad (5)$$

where  $G_{t+1}^R = \frac{R_{t+1}^h}{R_t^h}$  is gross rent growth. Equation (5) now explains the current price-rent

ratio in terms of expectations of future rent growth and future discount factors.

The non-linearity of equation (5) makes it difficult to empirically test. Following Cochrane (1992) we can derive a log-linear approximation to (5) using a Taylor expansion. Re-write (5) as

$$\frac{P_t^h}{R_t^h} = E_t \sum_{i=1}^{\infty} \exp\left(\sum_{k=1}^i (g_{t+k}^R - m_{t+k})\right) \quad (6)$$

where  $g_t^R = \log(G_t^R)$  and  $m_t = -\log(M_t)$ . Assuming that the growth rates of rent and the discount factors are stationary, we can take a first-order Taylor expansion of the right-hand side of (6) around  $E(g^R)$  and  $E(m)$ . This yields the following approximate present value relation for the price-rent ratio,

$$\frac{P_t^h}{R_t^h} \cong \frac{\Omega}{1-\Omega} + \frac{1}{1-\Omega} E_t \sum_{i=1}^{\infty} \Omega^i (\tilde{g}_{t+i}^R - \tilde{m}_{t+i}) \quad (7)$$

where  $\tilde{g}_t^R = g_t^R - E(g)$ ,  $\tilde{m}_t = m_t - E(m)$  and  $\Omega = \exp(E(g) - E(m))^2$ . The first term on the right-hand side of (7) indicates the value of the price-rent ratio in a world of certainty with constant rent growth and a constant discount rate<sup>3</sup>. The second right-hand side term explains variations in the price-rent ratio around its mean as due to changing expectations about rent growth and discount rates. The price-rent ratio increases when rent growth rates are forecast to rise (above their mean) and/or discount rates are forecast to decline. The approximate present-value relation (7) is what we use to try and explain the behaviour of price-rent ratios in the Sydney housing market.

A standard approach to testing present value models is to test the cross-equation restrictions imposed by the model (Campbell and Shiller, 1988). However previous studies of property prices suggest that formal tests of present value models will lead to their rejection by the data (Meese and Wallace, 1994; Clayton, 1996; Ayuso and Restoy, 2003). While this implies that there are systematic influences on price-rent ratios that are not accounted for in (7), formal tests do not necessarily provide much guidance as to the nature of the model's failure. Thus in examining if (7) can explain price-rent ratios in Sydney we focus (initially at least) on the weaker predictions of the model. Clearly if these weaker restrictions are not satisfied by the data then there is little hope for formal tests of cross-equation restrictions. An additional motivation for adopting this approach is

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<sup>2</sup> Cochrane (1992) uses a second-order Taylor expansion which would produce an additional term in (7) that measures covariance of rent growth with the discount rate. This would generate cross-sectional risk premiums, which are not the focus of this paper.

<sup>3</sup> This is just the classic Gordon (1962) growth model for stock prices.

that our data on house prices and rents are likely to contain considerable measurement errors and this could make formal statistical tests unreliable.

In the empirical analysis we focus on two basic implications of equation (7). The first of these simply follows from the fact that variations in the current price-rent ratio should reflect anticipated future changes in the growth of rents or in discount factors. If this is correct then the price-rent ratio must predict future growth in rents and/or the discount factor. Notice that if  $\tilde{g}_t^R$  and  $\tilde{m}_t$  are unpredictable then this test would fail, however in that case the price-rent ratio would just fluctuate randomly around its mean.

In deriving (7) the existence of a rational speculative bubble is ruled out by the imposition of a transversality condition. Therefore if we find that the price-rent ratio does not predict rent growth or some reasonable proxy for the stochastic discount factor, this could be taken as support for the view that speculative bubbles are an important influence on house prices.

Tests of predictive content provide evidence about what variables produce changes in the price-rent ratio. However they do not give us an estimate of the contribution to the variation in the price-rent ratio that a particular variable makes. For example how much of the variance of the price-rent ratio is due to predictable changes in rent growth. Cochrane (1992) provides a variance decomposition that can be used to answer such questions.

#### *A Variance Decomposition*

Multiply both sides of (7) by  $[\frac{P_t^h}{R_t^h} - E(\frac{P^h}{R^h})]$  and take unconditional expectations to get,

$$E[\frac{P_t^h}{R_t^h} - E(\frac{P^h}{R^h})]\frac{P_t^h}{R_t^h} \cong \frac{1}{1-\Omega} E\{[\frac{P_t^h}{R_t^h} - E(\frac{P^h}{R^h})]\sum_{i=1}^{\infty} \Omega^i (\tilde{g}_{t+i}^R - \tilde{m}_{t+i})\} \quad (8)$$

or

$$V(\frac{P_t^h}{R_t^h}) \cong \frac{1}{1-\Omega} \sum_{i=1}^{\infty} \Omega^i \left( \text{cov}(\frac{P_t^h}{R_t^h}, \tilde{g}_{t+i}^R) - \text{cov}(\frac{P_t^h}{R_t^h}, \tilde{m}_{t+i}) \right). \quad (9)$$

Equation (9) shows that the variance of the price-rent ratio can be approximated by covariances between the current price-rent ratio and future rent growth rates and the current price-rent ratio and future stochastic discount factors. Equation (9) can be made

operational by truncating the sum of the covariances at some point and replacing the population moments by their sample moments. The estimates provide a measure of the percentage of variation in the price-rent ratio that is due to rent growth relative to changes in the discount factor.

#### *Discount Factors*

One final issue concerns the measurement of the (unobserved) stochastic discount factor  $M_t$ . One choice is to set  $M_t = (1 + r_t^h)^{-1}$ , the ex-post real return on housing. This choice satisfies equation (1) by definition so that equation (7) now becomes an (approximate) identity (Campbell and Shiller, 1988). If we rule out speculative bubbles then it must be true that a rise in the price-rent ratio is associated with higher future rent growth and/or a fall in returns to housing. This suggests that using real housing returns in (7) and (9) is able to provide some evidence on the existence of speculative bubbles. For example if variation in rent growth and housing returns are found to contribute little to the variance of the price-rent ratio, the remaining candidate is a rational bubble. Thus current house prices are high only because buyers believe they will be even higher in the future.

Asset pricing theories suggest other choices for the stochastic discount factor<sup>4</sup>. For example the CAPM suggests the using the return to the market portfolio, while more traditional approaches to discounting suggest the use of the real interest rates. We employ three other choices for  $M_t$ , the real mortgage rate, the real bond rate and the real equity returns. These variables reflect returns on other assets and should influence the returns to and consequently the price of housing.

### **3. Data and Summary Statistics**

#### *Price-Rent Ratios*

We use quarterly data on median house prices and median weekly rents for Local Government Areas (LGAs) in metropolitan Sydney. The source of the price and rent data

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<sup>4</sup> In an optimal consumption framework  $M_t = \frac{\beta u'(c_{t+1})}{u'(c_t)}$  implying that we can approximate  $\tilde{m}_t$  by some

function of consumption growth (or possibly wealth growth). This is not feasible in the current context as we do not have data on consumption at the LGA level.

is the NSW Department of Housing's quarterly *Rent and Sales Reports*. Data on rents are derived from information provided when renters and landlords lodge their bonds with the Office of Fair Trading. Price data is based on title information provided by the Department of Lands. For most LGAs we have rent and price data for the period 1991:1 to 2006:3. In total we have data for 36 LGAs.

Data on rents is disaggregated into houses and units and also by the number of bedrooms. We construct three series for rents – for apartments, for houses and an aggregate rent series. When aggregating (for example to get overall rent for apartments from data on one and two bedroom apartments) we use weights from the 2001 Census - which indicates the number of one and two bedroom apartments in each LGA.

Data on sales prices are reported by housing title (strata and non-strata) and for aggregate sales. In constructing price-rent ratios sale prices for strata titles is used as the denominator for apartment rents, sale prices for non-strata titles for house rents and aggregate sale prices for aggregate rents.

Figure 2 shows aggregate price-rent ratios for three Sydney LGAs; Randwick, Manly and Penrith. Consistent with the aggregate Sydney data in Figure 1, price-rent ratios are relatively flat in the period 1991-96 after which there is steady growth until about 2001. There is acceleration in growth until 2003-04, after which price-rent ratios have declined, although generally not to pre-1996 levels. A broadly similar pattern for price-rent ratios is repeated across all Sydney LGAs.

#### *Other Series*

Mortgage rates are measured using the standard variable mortgage rate for banks, while the bond rate is the 10 year government bond rate. Stock returns are based on the S&P/ASX 200 accumulation index. To compute real rent growth and real returns we use the CPI for Sydney.

#### *Summary Statistics*

The full list of LGAs used in the study is provided in Table 1. In addition we report some summary statistics using aggregate data (i.e. houses and apartments) for each LGA. While our main interest is on the time series variation in the price-rent ratio it is informative to initially consider some features of the cross-section data.

Figure 3 plots average real housing returns against the minimum price-rent ratio for each LGA. Notice that there is a clear negative relationship – high price-rent ratios are associated with relatively low real returns.

Figure 4 plots real rent growth against the maximum price-rent ratio for each LGA. The results in this figure are indicative of our later findings using time series observations – there is relatively little relationship between rent growth and price-rent ratios.

The data in Table 1 can be used to provide an initial check on the accuracy of equation (7). Taking unconditional expectations of both sides of (7) gives,

$$E\left(\frac{P^h}{R^h}\right) \cong \frac{\Omega}{1 - \Omega} \quad (10)$$

where we can use the right-hand side to estimate the expected value of the price to rent ratio for each LGA. Recall that  $\Omega = \exp(E(g) - E(m))$ , so we estimate  $E(g)$  by mean real rent growth and  $E(m)$  by mean real bond rate. Table 1 reports the mean price-rent ratio and the estimate obtained from (10) for each LGA. Figure 5 plots these two series against each other. Ideally the points should lie on the 45 degree line. In fact the slope coefficient (1.26) is significantly above unity so there is a systematic tendency for the unconditional present value model to over-predict the expected price-rent ratio.

#### 4. What do Price-Rent Ratios Predict?

We now turn to the issue of whether observed movements in the price-rent ratios in the various LGAs of Sydney predict future changes in local rent growth and variations in future discount factors.

##### *Econometric Model*

The following regression model provides a simple framework for testing the predictive content of the price-rent ratio<sup>5</sup>;

$$x_t = \mu + \sum_{i=1}^k \alpha_i x_{t-i} + \sum_{i=1}^k \beta_i \frac{P_t^h}{R_t^h} + \omega_t \quad (11)$$

In equation (11)  $x$  indicates the variable that the present value model implies should be forecast by the price-rent ratio. Our five choices for  $x$  are; the growth rate of real rents,

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<sup>5</sup> The regression model also includes a dummy for introduction of the GST which takes the value 1 in 2000:3 and zero elsewhere and three seasonal dummy variables.

real returns to housing, the real mortgage rate, the real 10 year bond rate and real equity returns.

The present value model (7) predicts that if a particular choice of  $x$  is relevant we should find that lags of the price-rent ratio in (11) are jointly significant<sup>6</sup>. Furthermore the sign on  $\beta_i$  is predicted to be positive for real rent growth and negative for the variables measuring the stochastic discount factor. However rather than requiring each  $\beta_i$  to be a particular sign we focus on the sum of the ( $k$ )  $\beta_i$  s and its statistical significance. Finally we also compute the long-run or steady-state response of  $x$  to the price-rent ratio;  $\frac{\sum \beta_i}{1 - \sum \alpha_i}$ . These steady-state values indicate how useful the price-rent

ratio is at predicting long-term movements in  $x$ . In the steady-state (11) becomes

$$x = \frac{\mu}{1 - \sum \alpha_i} + \left[ \frac{\sum \beta_i}{1 - \sum \alpha_i} \right] \frac{P^h}{R^h} \quad (12)$$

so the slope coefficient measures the change in the long-run forecast long-run response of  $x$  to a (persistent) change in the price-rent ratio. When this number is close to zero the price-rent ratio must have relatively little long-run predictive power for  $x$ . Thus even if we observe a persistent rise in the price-rent ratio we would not predict a very large change in the future value of  $x$ .

### *Empirical Results*

The results presented in this section are for  $k=6$ . We have chosen a relatively large value for  $k$  to ensure that dynamics in the price-rent ratio series are adequately modelled and that the residuals are serially uncorrelated. Table 2 reports the p-value for an F-test of the joint significance of the  $\beta_i$  s, the sum of the estimated values of the  $\beta_i$  s and the t-statistics for the statistical significance of the sum. The LGAs are classified into 3 groups based on their distance to the Sydney CBD - an inner ring, middle ring and an outer ring.

In response to the question of which of the  $x$  variables are predicted by median price-rent ratios in the LGAs, we document four main findings.

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<sup>6</sup> Strictly speaking the price-rent ratio could predict rent growth and the discount factor but not Granger-cause them. This would arise where the only variables used by agents to predict rent growth and the discount factors were their own lags. One solution would be to follow the empirical literature on share prices and dividends and use long-horizon regressions. However in the current context we believe the use of an ADL model like (11) will produce more reliable results.

*Result #1: An increase in the price-rent ratio does not anticipate future increases in the growth rate of real rents.*

In about two thirds of LGAs the lags of the price-rent ratio are not statistically significant in explaining the growth rate of rents. In that third of LGAs where lags of the price-rent ratio are jointly significant, the sum of the estimated coefficients on the lags on the price-rent ratio is almost always statistically significant and of the wrong sign. There is really only one LGA (Fairfield) which could be said to provide reasonable statistical support for the present value model (7) with respect to rents.

Cochrane (1992) emphasises that it is only predictable changes in real rent growth that should produce variation in the price-rent ratio. If real rent growth is unpredictable its future behaviour will not affect current house prices and we should not find evidence of Granger-causality. Thus one explanation for our results is that the growth rate of real rents is not predicable. This does not seem to be the case. We estimate an AR(6) model for real rent growth in each LGA and test the joint significance of the six lags. Figure 6 shows the p-value associated with the relevant F-test. For most LGAs the growth rate of rent is predictable by its own lags. Since rent growth is predicable it should produce variations in the price-rent ratio – our Granger-causality tests suggest it does not in two-thirds of LGAs and in the remaining one-third any effect is generally of the wrong sign.

Where houses and apartments are owner-occupied there is no explicit rental payment, rather there will be an unobserved utility flow to the owner-occupier of the house. It is conceivable that market rents may provide an imperfect proxy for the flow of services to owner-occupied housing. It is difficult to formally test this possibility in the current situation, although to explain the behaviour of price-rent ratios in Sydney growth of utility flows from home ownership must have been expected to grow much faster than rents.

*Result #2: An increase in the price-rent ratio does predict declining real returns to housing.*

When real housing returns are used in (11) we are testing an identity rather than a specific asset pricing model. It is therefore reassuring to find that an increase in the price-rent ratio forecasts declining real returns to housing in all LGAs. These findings are consistent with the cross-section averages in Figure 1. The economic content to this

finding arises from the fact that equation (7) is derived from imposing a no-bubbles condition. Thus if price-rent ratios did not predict either rents or housing returns it would have suggested a possible role for bubbles in determining house prices. The finding that price-rent ratios do forecast returns is evidence against the speculative bubble explanation for Sydney property prices.

*Result #3: An increase in the price-rent ratio predicts declining real interest rates.*

Across virtually all LGAs we find strong evidence that price-rent ratios are useful in forecasting future movements in the (ex-post) real mortgage rate and the 10 year bond rate. Furthermore the sign of the effect is consistent with our theory - increases in price-rent ratios anticipate future declines in real interest rates. Based on the magnitude of the t-statistics the price-rent ratio seems to be slightly better at forecasting the real bond rate than the real variable mortgage rate.

*Result #4: An increase in the price-rent ratio does not predict declining real equity returns.*

For a majority of LGAs there is evidence that price-rent ratios do forecast real equity returns. However it is also evident from Table 1 that there is no systematic pattern to the sum of the estimated coefficients on lags of the price-rent ratio. In addition the sum is never statistically significant.

What emerges from the prediction tests is that Sydney property prices appear to respond to anticipated future movements in long-term real interest rates. This raises the question as to whether there are systematic differences across LGAs in the response of price-rent ratios to changes in the long-term interest rates. If property prices in some locations responded more to expected changes in real rates this could explain differences in the level of house prices across Sydney. We examine this by looking at the magnitude of the long-run coefficients on the price-rent ratio in the model for the real bond rate.

*What are the Long-Run Effects?*

The (absolute value) of the long-run coefficients are reported in Figure 7 for each LGA. With a few exceptions there are not large differences across LGAs in the estimated long-run coefficients. Coefficient estimates appear to be somewhat above average for five LGAs - North Sydney, Lane Cove, Willoughby, Strathfield and Ashfield. The average effect across Sydney is 0.016. This implies that a rise in the quarterly price-rent ratio by

10 percentage points (from 90 to 100) predicts a decline in future real bonds rates of about 0.16 percentage points. Reversing this - if the RBA could engineer a credible expected fall in real bond rates of 0.50 basis points – this would be associated with an increase in the quarterly price-rent ratio by about 30 percentage points (or in the annual price-rent ratio by about 7.5 percentage points).

Further evidence of how well the price-rent ratios predict real bond rates in the long-run is given in Figure 8. This graph plots the long-run predictions for the real bond rate using equation (11) and price-rent ratios in three LGAs - Randwick, Manly and Penrith against the actual value of the real bond rate. While the real bond rate is more variable than its predictions, the predicted values do track the downward trend in the real bond rate over the sample period.

#### *Decomposing the Variance of the Price-Rent Ratio*

To measure the relative contributions of the  $x$  variables make to the variance of the price-rent ratio we can use the decomposition given in equation (9). Given the results of the prediction tests, Table 3 reports the percentage of the variance of price-rent ratios in Sydney LGAs that is accounted for by rent growth, real returns and the real bond rate. The first column shows the contribution from real rent growth. The estimated contribution of real rent growth to the variance of the price-rent ratio is typically small and frequently negative. This is consistent with our earlier finding that a rise in the price-rent ratio does not predict higher growth in real rents. The negative numbers (to the extent they are significant) imply that rises in the price-rent ratio are actually predicting lower real rent growth.

When real returns to housing are used in equation (9) we find a much greater proportion of the variance of the price-rent ratio is accounted for; with the highest proportion (123 percent) for Woollahra and the lowest for Penrith (40 percent). In light of the low or negative contributions of real rent growth we expect the contribution of real returns to be 100 percent or (even higher). While this is the case for some LGAs, it is notable that in some of the outer western Sydney LGAs (Penrith, Liverpool, Campbelltown) the contribution from variation in returns is less than 50 percent. In general there is some tendency for the contribution of real returns to vary with distance from the CBD, with the outer LGAs having a lower contribution. For the inner ring of

LGAs real returns explain around 90 percent of the variance of price-rent ratios. This figure falls to about 58 percent for the middle ring and to 52 percent for the outer ring of LGAs.

The results for the outer ring of LGAs are interesting. What they suggest is that in these outer ring of LGAs a reasonably large proportion of the variation in price-rent ratios cannot be explained by future rent growth or even by future housing returns. According to the present value relation the only other possible source of variation in the price-rent ratio is a rational bubble. This suggests that at least some part of the increase in house prices in western Sydney were due to a (rational) expectation that future house prices would be higher; independent of real rent growth and the behaviour of discount rates.

The final column shows the contribution of the real bond rate to the variance of the price-rent ratio. Across LGAs the contributions range from 4 to 23 percent. In general the contributions tend to be higher for inner ring LGAs. Nevertheless the decomposition indicates that less than a quarter of the variance in the price-rent ratio can be attributed to changing expectations about future real interest rates.

#### *Are There Differences Between Houses and Apartments?*

It is sometimes suggested that an important element of the Sydney property market is investment in rental properties, in particular apartments. A common belief is that during the property boom, investors were prepared to buy investment properties for the anticipated capital gains rather than rental considerations. Some evidence of differences between houses and apartments can be seen from Figure 9 which compares the estimates of the long-run coefficient on the price-rent ratio. For LGAs in the inner and middle rings apartment prices appear to be more responsive to anticipated changes in the real interest rate than are house prices.

### **5. Conclusions**

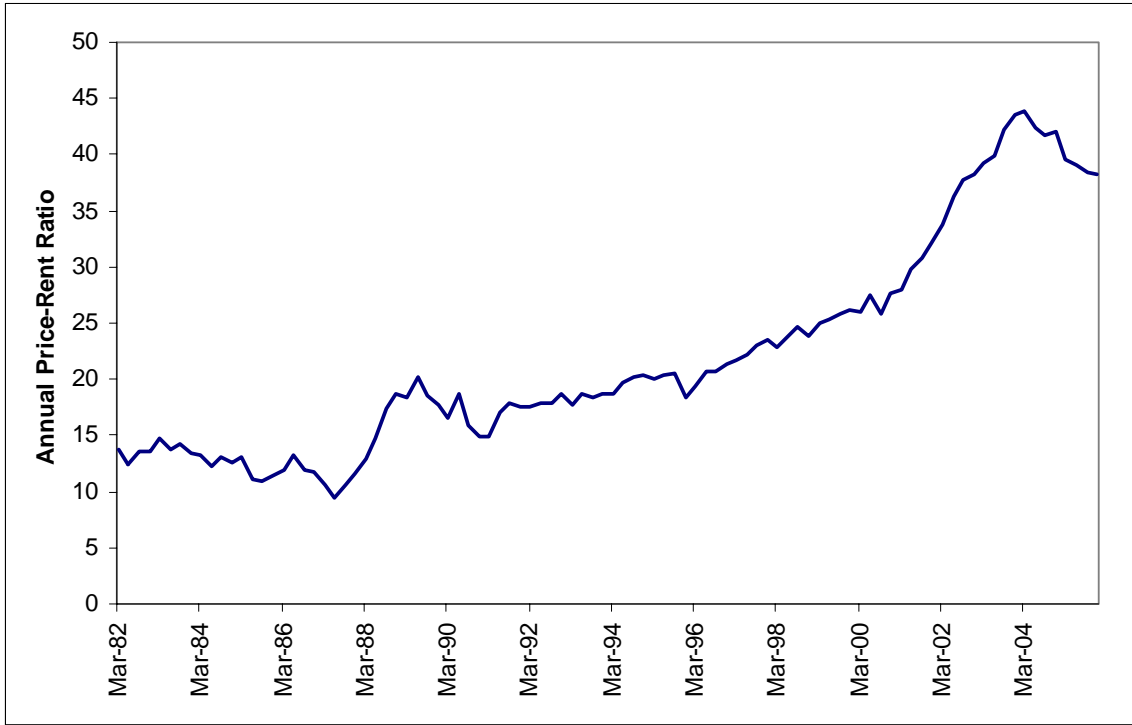
The results in this paper suggest that a partial explanation of the recent boom in Sydney property prices lies with lower long-term real interest rates. During the second half of the 1990s it appears that buyers and sellers of residential property came to expect a persistent fall in real interest rates. This led to an upward re-valuation of the flow of rents (or utility services) associated with houses and apartments and drove up market prices (relative to rents). At the same time there seems to be little evidence that buyers

and sellers were expecting any (short or medium term) increase in the growth rate of rents. Just as future dividend or earnings growth apparently explains little of the variation in stock prices, we find that real rent growth explains little of the variation in residential property prices.

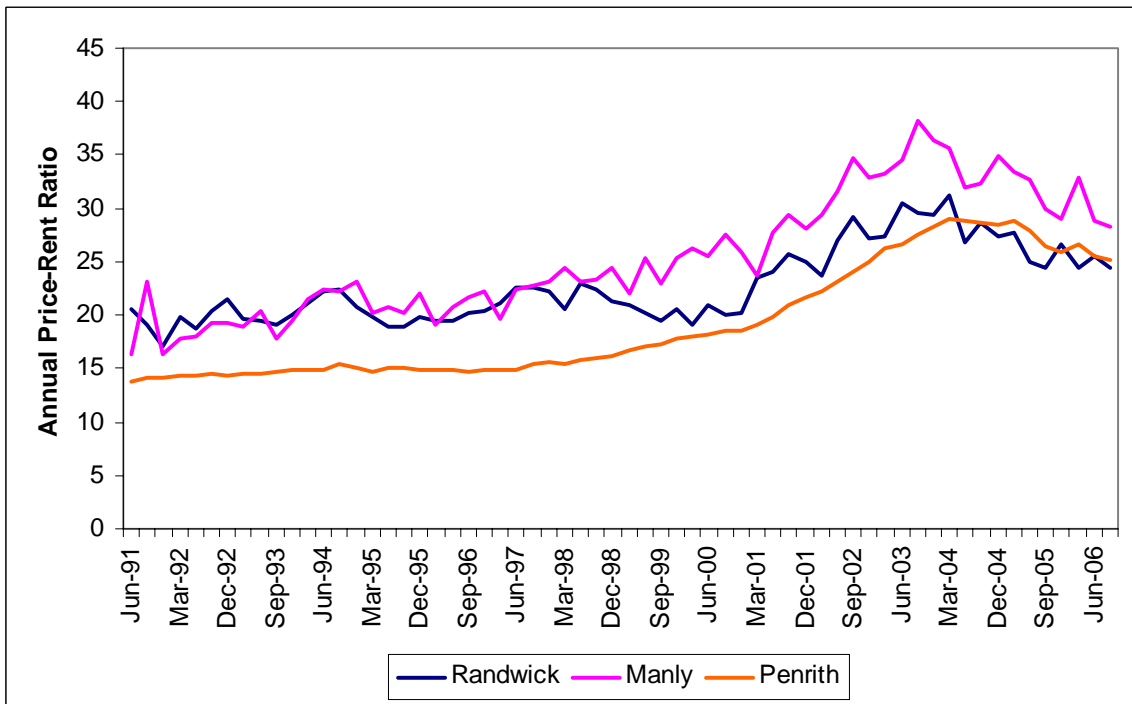
The idea that expected real interest rate changes drive house prices is a simple and theoretically appealing story. Unfortunately some of our empirical results also suggest that it is incomplete. At most only about a quarter of the variation in Sydney price-rent ratios seems to be due to expected variation in real interest rates. In explaining the remaining variation we find that our story varies with location. For properties that are close to the CBD almost all of the variation in the price-rent ratio can be explained by expected variation in future returns to housing. An economic interpretation of this finding is that there was a decline on the risk premium on houses and apartments leading to a bidding-up of the property prices and a lowering of expected (excess) returns. While this conclusion leaves unanswered the question of what drives the risk premium, at least it rules out any influence on house prices from speculative bubbles.

Finally when we consider property in the outer western suburbs of Sydney even appealing to a declining risk premium leaves about half of the variation in price-rent ratios unaccounted for. According to our model the only remaining explanation is a rational speculative bubble.

**Figure 1: Median Price-Rent Ratio for Established Houses in Sydney (1982-2005)**



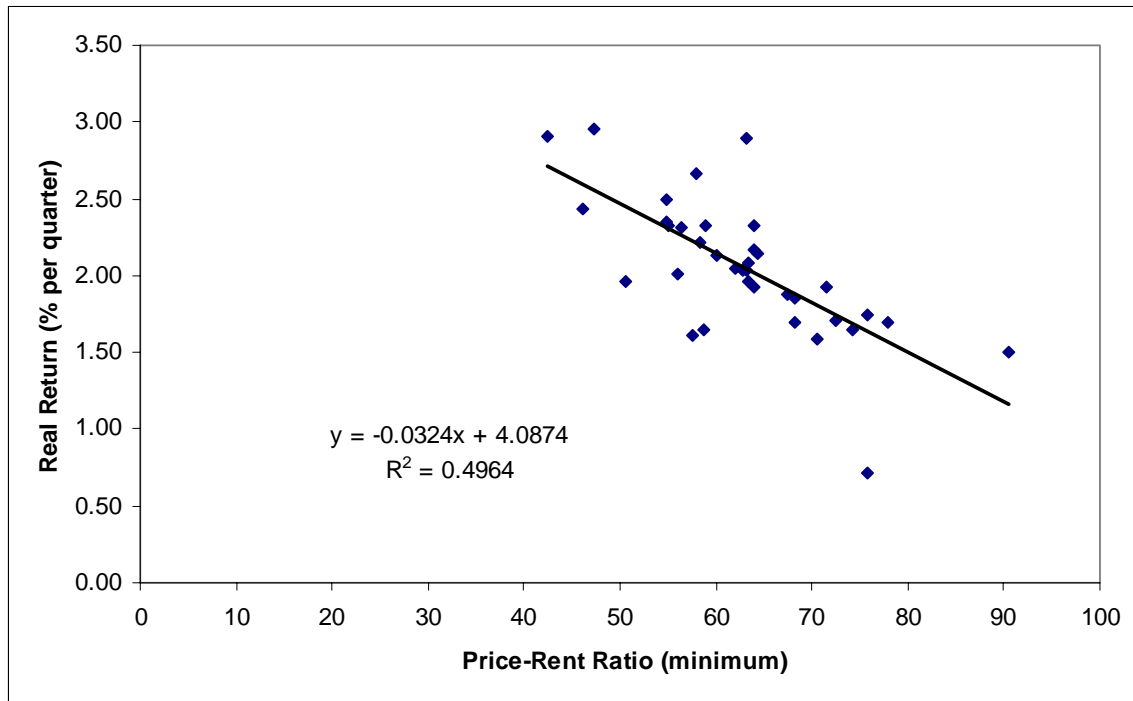
**Figure 2: Median Price-Rent Ratios in 3 Sydney LGAs (1991-2006)**



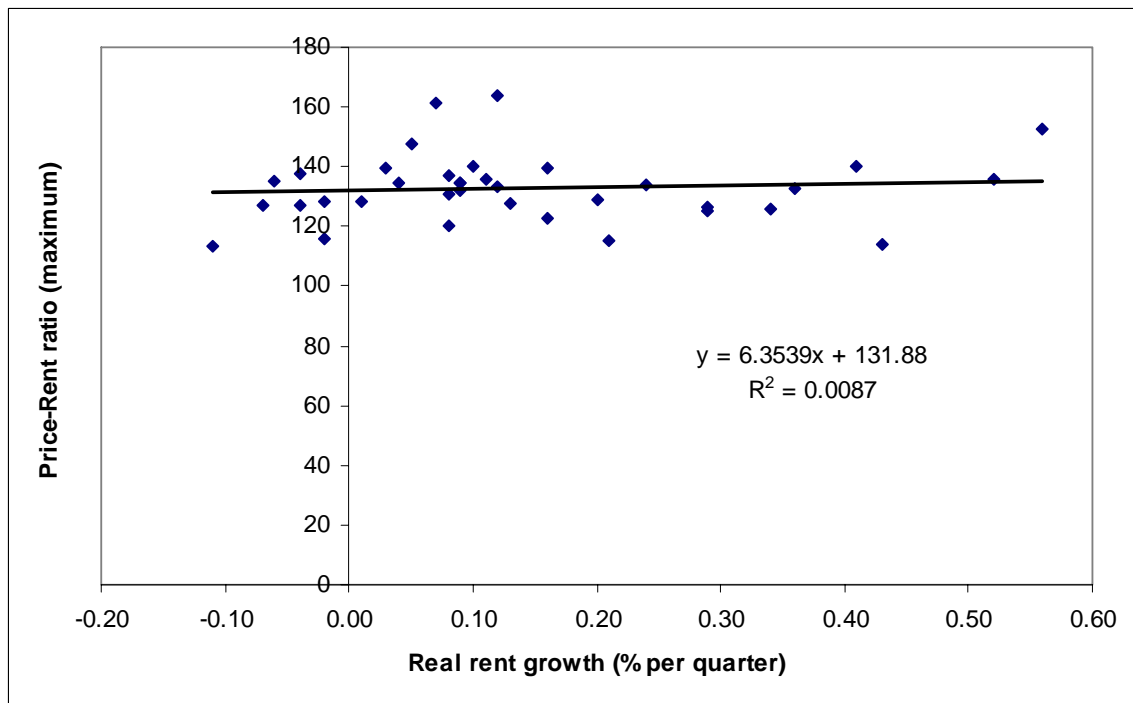
**Table 1: Summary Statistics for LGAs in Sydney**

LGA	Sample	Price-Rent Ratio (Quarterly)				Real Rent Growth	Real Return
		Min	Max	Mean	Predicted	% per qtr	% per qrt
Ashfield	91:1-06:3	55.0	115.4	79.9	118.3	0.21	2.33
Botany	91:1-06:3	50.6	128.9	84.2	116.9	0.20	1.96
Lane Cove	91:3-06:3	57.6	122.7	87.9	111.7	0.16	1.61
Leichhardt	91:2-06:3	64.0	140.1	92.4	155.2	0.41	2.33
Marrickville	91:1-06:3	47.3	132.5	86.4	144.0	0.36	2.96
North Sydney	91:3-06:3	70.5	114.3	91.0	160.2	0.43	1.59
Randwick	91:2-06:3	68.3	125.1	90.5	130.8	0.29	1.70
Waverley	91:1-06:3	63.2	135.8	89.6	187.2	0.52	2.04
Woollahra	91:1-06:3	75.9	163.5	113.1	106.9	0.12	0.71
Auburn	91:1-06:3	59.0	139.8	90.5	111.7	0.16	2.32
Bankstown	91:1-06:3	62.8	137.9	89.2	91.1	-0.04	2.04
Burwood	92:4-06:3	71.4	134.1	95.3	122.7	0.24	1.92
Canterbury	91:2-06:3	56.0	132.1	83.2	103.5	0.09	2.01
Hurstville	91:2-06:3	67.4	139.3	93.7	97.4	0.03	1.88
Kogarah	91:2-06:3	72.5	134.8	92.7	103.5	0.09	1.71
Ku-ring-gai	91:3-06:3	90.4	161.5	117.0	101.4	0.07	1.50
Manly	91:1-06:3	58.0	152.9	101.5	202.3	0.56	2.66
Parramatta	91:1-06:3	63.3	128.6	87.0	92.8	-0.02	1.96
Rockdale	91:2-06:3	64.0	127.9	85.4	108.0	0.13	1.92
Ryde	91:1-06:3	63.3	139.9	92.6	104.6	0.10	2.08
Strathfield	91:1-06:3	58.7	120.0	93.5	102.4	0.08	1.65
Willoughby	94:3-06:3	75.9	126.6	95.5	130.8	0.29	1.75
Baulkham Hills	91:1-06:3	77.9	147.3	100.8	99.4	0.05	1.69
Blacktown	91:1-06:3	56.3	127.0	78.9	88.7	-0.07	2.31
Blue Mountains	91:1-06:3	64.3	136.0	88.7	105.7	0.11	2.14
Campbelltown	91:1-06:3	46.1	113.6	72.3	85.6	-0.11	2.43
Fairfield	91:1-06:3	60.0	135.2	83.4	89.5	-0.06	2.13
Gosford	91:1-06:3	64.0	134.3	89.3	103.5	0.09	2.17
Hawkesbury	91:1-06:3	58.4	127.3	83.6	91.1	-0.04	2.22
Holroyd	91:1-06:3	62.0	128.3	85.1	95.5	0.01	2.05
Hornsby	91:1-06:3	74.2	130.8	94.5	102.4	0.08	1.65
Liverpool	91:2-06:3	42.5	136.9	83.7	102.4	0.08	2.91
Penrith	91:1-06:3	54.8	116.2	76.4	92.8	-0.02	2.35
Sutherland	91:1-06:3	68.3	133.0	91.6	106.9	0.12	1.85
Warringah	91:3-06:3	54.8	126.0	89.3	139.9	0.34	2.50
Wyong	91:2-06:3	63.2	134.8	83.5	98.4	0.04	2.90

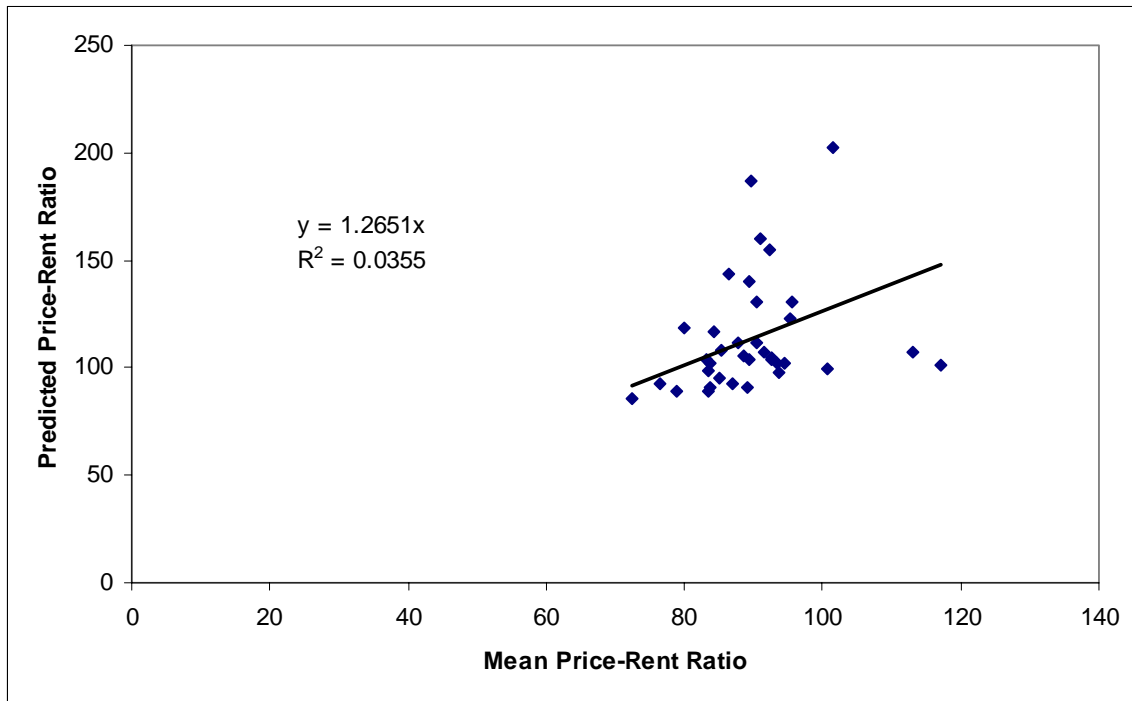
**Figure 3: Scatter-Plot of Real Returns vs. Price-Rent Ratio**



**Figure 4: Scatter-Plot of Real Rent Growth vs. Price-Rent Ratio**



**Figure 5: Scatter-Plot of Mean and Predicted Price-Rent Ratios**



**Table 2: What do House Price-to-Rent Ratios Predict?****Dependent Variable**

<b>LGA (Inner Ring)</b>	<b>Real Rent Growth</b>	<b>Real Housing Returns</b>	<b>Real Mortgage Rate</b>	<b>Real Bond Rate</b>	<b>Real Stock Returns</b>
<i>Ashfield</i>					
Joint Sig. (p-value)	0.1594	0.0177	0.0008	0.0002	0.1542
Coeff. Sum	-0.1035	-0.0933	-0.0114	-0.0153	-0.0083
t-statistic	(-0.763)	(-1.303)	(-2.766)	(-3.953)	(-0.236)
<i>Botany</i>					
Joint Sig. (p-value)	0.3464	0.2060	0.5057	0.0181	0.0000
Coeff. Sum	-0.0139	-0.0841	-0.0062	-0.0092	0.0055
t-statistic	(-1.090)	(-1.828)	(-2.013)	(-3.095)	(0.251)
<i>Lane Cove</i>					
Joint Sig. (p-value)	0.0012	0.0020	0.0358	0.0002	0.3074
Coeff. Sum	-0.0279	-0.2819	-0.0115	-0.0135	0.0006
t-statistic	(-1.137)	(-2.914)	(-2.581)	(-3.327)	(0.016)
<i>Leichhardt</i>					
Joint Sig. (p-value)	0.0350	0.0238	0.1029	0.0004	0.1136
Coeff. Sum	-0.0177	-0.0689	-0.0078	-0.0107	0.0039
t-statistic	(-1.387)	(-2.331)	(-2.529)	(-3.648)	(0.137)
<i>Marrickville</i>					
Joint Sig. (p-value)	0.5777	0.0143	0.2375	0.0185	0.1444
Coeff. Sum	0.0026	-0.1000	-0.0066	-0.0092	-0.0165
t-statistic	(0.204)	(-2.903)	(-2.389)	(-3.501)	(-0.250)
<i>North Sydney</i>					
Joint Sig. (p-value)	0.2337	0.0003	0.1086	0.1086	0.0312
Coeff. Sum	-0.0006	-0.2192	-0.0151	-0.0184	-0.0165
t-statistic	(-0.018)	(-2.952)	(-2.578)	(-3.231)	(-0.323)
<i>Randwick</i>					
Joint Sig. (p-value)	0.0047	0.0188	0.1245	0.0035	0.0028
Coeff. Sum	-0.0148	-0.1810	-0.0084	-0.0111	0.0186
t-statistic	(-0.786)	(-2.656)	(-2.136)	(-3.021)	(0.531)
<i>Waverley</i>					
Joint Sig. (p-value)	0.0027	0.0615	0.1017	0.0334	0.1679
Coeff. Sum	-0.0661	-0.1753	-0.0087	-0.0114	0.0021
t-statistic	(-2.774)	(-2.830)	(-2.791)	(-3.612)	(0.070)
<i>Woollahra</i>					
Joint Sig. (p-value)	0.0394	0.0002	0.1692	0.0084	0.5145
Coeff. Sum	-0.0180	-0.1450	-0.0050	-0.0070	0.0107
t-statistic	(-0.480)	(-3.933)	(-2.117)	(-3.161)	(0.425)

Table 2: continued

## Dependent Variable

LGA (Middle Ring)	Real Rent Growth	Real Housing Returns	Real Mortgage Rate	Real Bond Rate	Real Stock Returns
<i>Auburn</i>					
Joint Sig. (p-value)	0.4482	0.0001	0.0912	0.0047	0.5368
Coeff. Sum	0.0066	-0.0456	-0.0067	-0.0086	-0.0024
t-statistic	(0.999)	(-2.289)	(-2.524)	(-3.164)	(-0.110)
<i>Bankstown</i>					
Joint Sig. (p-value)	0.0543	0.0000	0.0142	0.0057	0.0000
Coeff. Sum	0.0047	-0.0163	-0.0066	-0.0088	0.0086
t-statistic	(0.541)	(-1.884)	(-2.376)	(-2.859)	(0.466)
<i>Burwood</i>					
Joint Sig. (p-value)	0.8311	0.0011	0.2547	0.1852	0.0041
Coeff. Sum	-0.0150	-0.1830	-0.0071	-0.0098	0.0515
t-statistic	(-0.630)	(-2.705)	(-1.937)	(-2.589)	(1.687)
<i>Canterbury</i>					
Joint Sig. (p-value)	0.8410	0.0002	0.0007	0.0000	0.0018
Coeff. Sum	0.0012	-0.0858	-0.0080	-0.0114	0.0113
t-statistic	(0.102)	(-2.186)	(-2.370)	(-3.550)	(0.424)
<i>Hurstville</i>					
Joint Sig. (p-value)	0.8978	0.1978	0.2069	0.0525	0.1624
Coeff. Sum	-0.0050	-0.0790	-0.0060	-0.0089	0.0249
t-statistic	(-0.415)	(-2.641)	(-2.249)	(-3.204)	(1.003)
<i>Kogarah</i>					
Joint Sig. (p-value)	0.1022	0.0298	0.5754	0.1098	0.0000
Coeff. Sum	0.0053	-0.1341	-0.0069	-0.0108	0.0224
t-statistic	(0.223)	(-2.593)	(-1.753)	(-2.801)	(0.820)
<i>Ku-ring-gai</i>					
Joint Sig. (p-value)	0.5982	0.0001	0.0209	0.0002	0.1964
Coeff. Sum	-0.0032	-0.0562	-0.0043	-0.0065	0.0295
t-statistic	(-0.191)	(-2.116)	(-1.642)	(-2.470)	(1.153)
<i>Manly</i>					
Joint Sig. (p-value)	0.8806	0.0145	0.0059	0.0037	0.0015
Coeff. Sum	-0.0076	-0.0938	-0.0071	-0.0099	-0.0018
t-statistic	(-0.404)	(-2.386)	(-2.469)	(-3.544)	(-0.085)
<i>Parramatta</i>					
Joint Sig. (p-value)	0.3786	0.0132	0.1708	0.0228	0.0545
Coeff. Sum	-0.0001	-0.0456	-0.0068	-0.0092	0.0118
t-statistic	(-0.016)	(-2.161)	(-2.327)	(-3.188)	(0.471)
<i>Rockdale</i>					
Joint Sig. (p-value)	0.0136	0.0873	0.0054	0.0036	0.3207
Coeff. Sum	-0.0031	-0.0654	-0.0076	-0.0105	-0.0012
t-statistic	(-0.239)	(-2.099)	(-2.217)	(-3.080)	(-0.036)
<i>Ryde</i>					
Joint Sig. (p-value)	0.1772	0.3480	0.0053	0.0030	0.0034
Coeff. Sum	-0.0016	-0.0684	-0.0062	-0.0096	0.0169
t-statistic	(-0.233)	(-2.111)	(-2.201)	(-3.448)	(0.718)

<i>Strathfield</i>					
Joint Sig. (p-value)	0.0000	0.0000	0.3356	0.1029	0.9831
Coeff. Sum	0.0400	-0.7802	-0.0082	-0.0087	-0.0565
t-statistic	(0.086)	(-5.190)	(-1.326)	(-1.346)	(-0.665)
<i>Willoughby</i>					
Joint Sig. (p-value)	0.9296	0.0027	0.0271	0.0231	0.0128
Coeff. Sum	-0.0245	-0.1711	-0.0073	-0.0120	0.0768
t-statistic	(-0.711)	(-1.976)	(-1.865)	(-3.044)	(1.555)

**Table 2 continued**

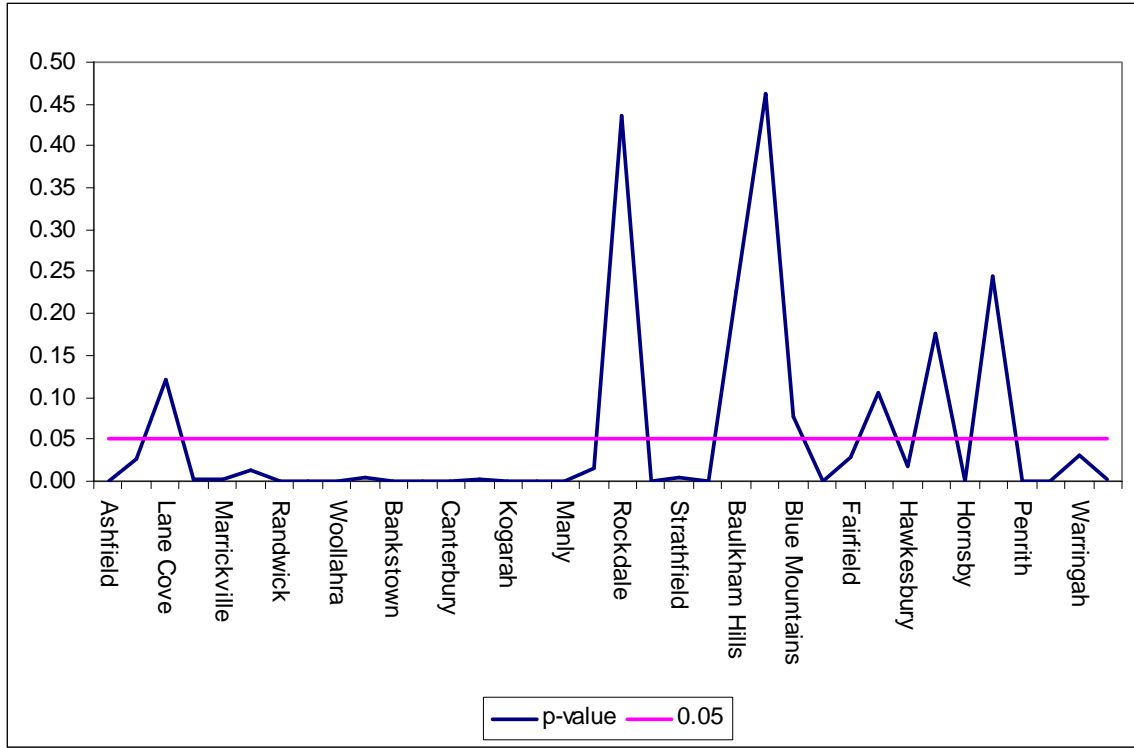
**Dependent Variable**

<b>LGA (Outer Ring)</b>	<b>Real Rent Growth</b>	<b>Real Housing Returns</b>	<b>Real Mortgage Rate</b>	<b>Real Bond Rate</b>	<b>Real Stock Returns</b>
<i>Baulkham Hills</i>					
Joint Sig. (p-value)	0.0305	0.0016	0.4677	0.0828	0.0019
Coeff. Sum	0.0031	-0.0386	-0.0048	-0.0071	0.0075
t-statistic	(0.394)	(-2.374)	(-1.794)	(-2.599)	(0.309)
<i>Blacktown</i>					
Joint Sig. (p-value)	0.1518	0.0596	0.1584	0.0067	0.0493
Coeff. Sum	0.0014	-0.0274	-0.0061	-0.0091	0.0241
t-statistic	(0.192)	(-1.607)	(-2.236)	(-3.368)	(1.004)
<i>Blue Mountains</i>					
Joint Sig. (p-value)	0.0012	0.0156	0.0269	0.0006	0.1294
Coeff. Sum	-0.0018	-0.0204	-0.0051	-0.0081	0.0228
t-statistic	(-0.214)	(-1.537)	(-2.149)	(-3.523)	(0.858)
<i>Campbelltown</i>					
Joint Sig. (p-value)	0.0809	0.0092	0.0294	0.0392	0.0197
Coeff. Sum	0.0109	-0.0403	-0.0077	-0.0103	0.0122
t-statistic	(0.931)	(-2.825)	(-2.786)	(-3.363)	(0.455)
<i>Fairfield</i>					
Joint Sig. (p-value)	0.0542	0.0594	0.3552	0.0335	0.0000
Coeff. Sum	0.0135	-0.0289	-0.0054	-0.0084	0.0071
t-statistic	(2.034)	(-2.463)	(-2.141)	(-3.477)	(0.374)
<i>Gosford</i>					
Joint Sig. (p-value)	0.1230	0.0055	0.0626	0.0286	0.0023
Coeff. Sum	0.0013	-0.0237	-0.0073	-0.0098	0.0112
t-statistic	(0.158)	(-1.959)	(-2.468)	(-3.242)	(0.497)
<i>Hawkesbury</i>					
Joint Sig. (p-value)	0.0000	0.5185	0.0917	0.0081	0.0020
Coeff. Sum	0.0131	-0.0324	-0.0065	-0.0093	-0.0002
t-statistic	(1.734)	(-1.707)	(-2.225)	(-3.236)	(-0.009)
<i>Holroyd</i>					
Joint Sig. (p-value)	0.7646	0.1067	0.0166	0.0010	0.1998
Coeff. Sum	-0.0038	-0.0420	-0.0075	-0.0101	0.0177
t-statistic	(-0.481)	(-1.702)	(-2.682)	(-3.738)	(0.672)

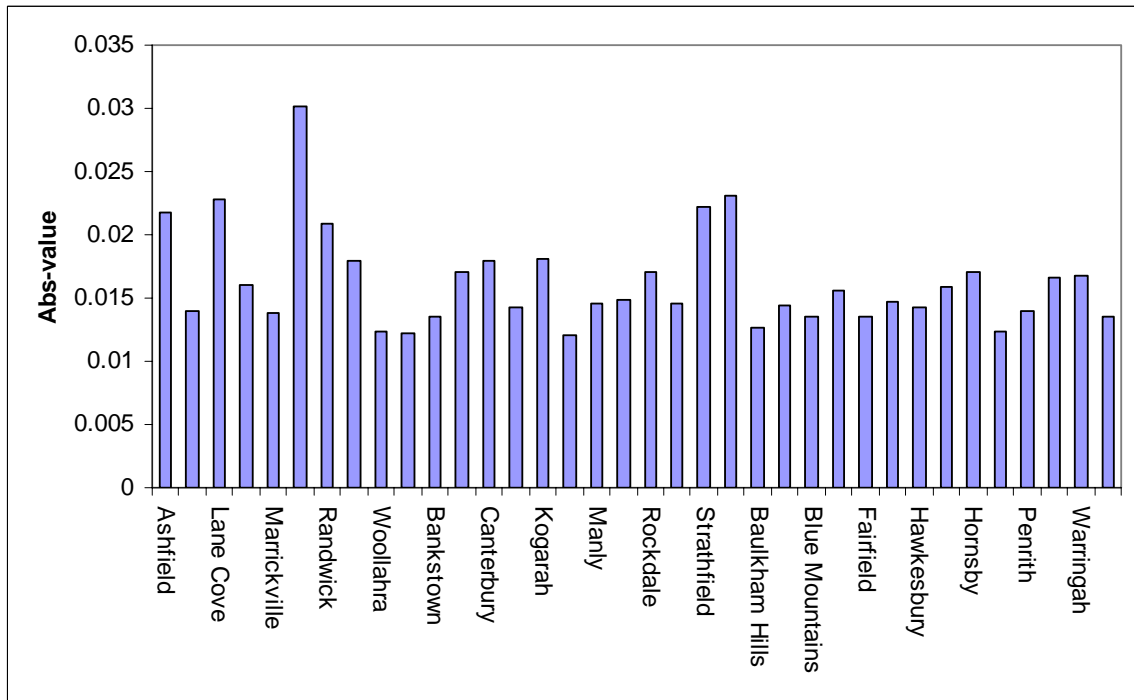
<i>Hornsby</i>					
Joint Sig. (p-value)	0.0338	0.0273	0.0000	0.0000	0.0226
Coeff. Sum	-0.0071	-0.0421	-0.0074	-0.0109	0.0191
t-statistic	(-0.649)	(-1.587)	(-2.221)	(-3.014)	(0.642)
<i>Liverpool</i>					
Joint Sig. (p-value)	0.9512	0.3283	0.0015	0.0003	0.0378
Coeff. Sum	-0.0008	-0.0353	-0.0066	-0.0093	0.0104
t-statistic	(-0.096)	(-1.816)	(-2.943)	(-4.135)	(0.472)
<i>Penrith</i>					
Joint Sig. (p-value)	0.1639	0.0014	0.5342	0.0890	0.0092
Coeff. Sum	0.0103	-0.0068	-0.0072	-0.0090	0.0182
t-statistic	(1.200)	(-0.564)	(-1.972)	(-2.425)	(0.627)
<i>Sutherland</i>					
Joint Sig. (p-value)	0.7323	0.1277	0.2505	0.0105	0.0002
Coeff. Sum	-0.0090	-0.0457	-0.0069	-0.0110	-0.0022
t-statistic	(-1.278)	(-2.122)	(-2.194)	(-3.552)	(-0.089)
<i>Warringah</i>					
Joint Sig. (p-value)	0.0533	0.1012	0.3770	0.0112	0.0034
Coeff. Sum	-0.0102	-0.0913	-0.0073	-0.0100	0.0007
t-statistic	(-1.096)	(-1.820)	(-2.160)	(-3.110)	(0.027)
<i>Wyang</i>					
Joint Sig. (p-value)	0.0000	0.1218	0.0000	0.0000	0.0000
Coeff. Sum	0.0011	-0.0158	-0.0045	-0.0071	0.0119
t-statistic	(0.147)	(-1.068)	(-1.801)	(-2.887)	(0.471)

**Notes: Each model includes 6 lags of the dependent variable and the house price-to-rent ratio. The above numbers indicate the results of an F-test for the joint significance of the lags of the price-to-rent ratio, the sum of the coefficients on the lags of the price-to-rent ratio and its t-statistic. White's (1980) HCCM estimator is used.**

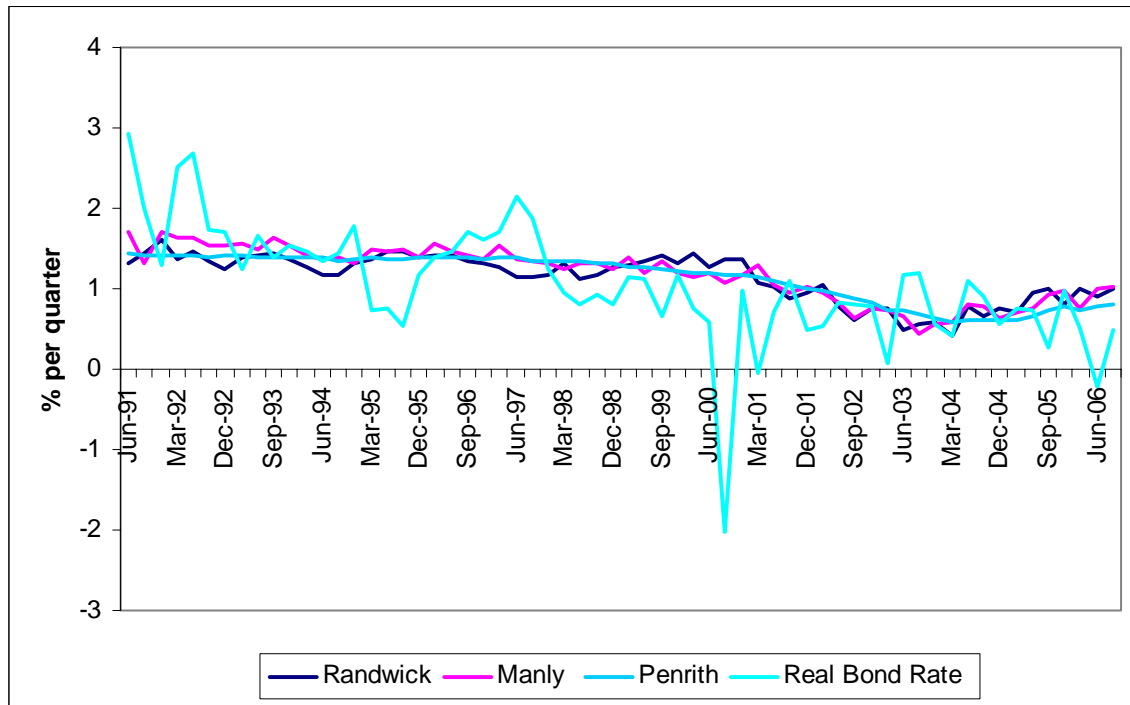
**Figure 6: Predicability of Real Rent Growth in LGAs**



**Figure 7: Long-Run Marginal Effect of the Price-Rent Ratio on the Real Bond Rate**



**Figure 8: Prediction of the Real Bond Rate Based on Long-Run Estimates for Three LGAs.**



**Table 3: Variance Decomposition of the Price-Rent Ratio**

**Contribution to Variance (percent)**

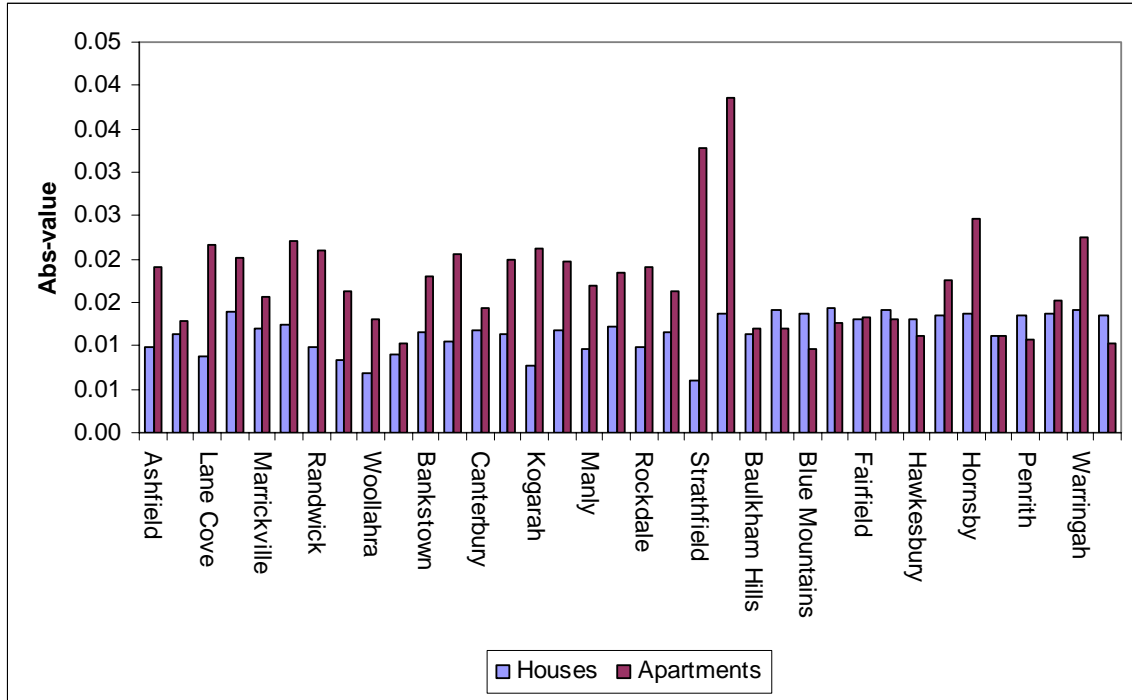
LGA	Real Rent Growth	Real Returns	Real Bond Rate
Ashfield	-12.9	65.1	16.8
Botany	-12.8	86.9	13.9
Lane Cove	-22.3	104.6	15.7
Leichhardt	-16.9	75.7	15.3
Marrickville	-9.3	50.0	12.0
North Sydney	-16.6	99.4	23.1
Randwick	-25.7	108.7	15.6
Waverley	-26.6	96.9	15.6
Woollahra	-32.4	123.1	12.9
Auburn	6.8	53.9	12.8
Bankstown	1.5	42.7	10.5
Burwood	-19.1	86.9	10.7

Canterbury	0.9	66.3	12.7
Hurstville	-6.6	56.9	12.4
Kogarah	-1.9	79.9	18.1
Kur-ring-gai	-12.3	60.4	9.4
Manly	-6.2	64.2	14.1
Parramatta	-4.1	61.4	12.5
Rockdale	-7.7	72.8	15.7
Ryde	-12.8	63.0	12.4
Strathfield	5.1	83.7	14.6
Willoughby	-1.6	92.7	3.7
Baulkham Hills	2.1	50.0	10.5
Blacktown	-3.2	46.0	8.2
Blue Mountains	-1.2	42.6	9.7
Campbelltown	7.4	41.6	8.5
Fairfield	7.3	47.5	8.1
Gosford	-1.1	42.8	10.9
Hawkesbury	-1.2	47.3	11.1
Holroyd	-6.8	58.4	11.3
Hornsby	-23.0	79.9	13.0
Liverpool	0	40.2	10.1
Penrith	2.9	39.3	9.3
Sutherland	-11.8	65.5	13.5
Warringah	-13.5	76.5	17.4
Wyong	-1.9	47.6	8.4

**Notes:** Entries are the percent of the variance of the log price-rent ratio that is attributable to rent growth, return and real interest rate forecasts. They are computed as

$$100 \times \left[ \sum_{i=1}^{20} \text{cov} \left( \ln \frac{P_t^h}{R_t^h}, x_{t+i} \right) \right] \left[ \text{var} \left( \ln \frac{P_t^h}{R_t^h} \right) \right]^{-1}.$$

**Figure 9: Long-Run Marginal Effects of the Price-Rent Ratio on the Real Bond Rate for Houses and Apartments**



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